SOME EULER-GRAPHS FROM EULER DIAGRAM

Manabjyoti Choudhury*

Bichitra Kalita**

ABSTRACT:

In this paper, some properties and theoretical results of the Euler Graph G(2n+10, 4n+20) for n ≥1, obtained from a special pattern of Euler Diagrams have been studied. In addition to this the various properties related to the dual graph H(2n+12, 4n+20) and the intersection graph G′(2n+11, 2n+18)of the graph G(2n+10, 4n+20) for n ≥1, have also been focused.

KEYWORDS: Euler diagram, Euler graph, Intersection graph, Dual graph, Colorability.

* Computer Science and Engineering Department, Assam down town University, Assam, India

** Department of Computer Applications, Assam Engineering College, Gauhati University, Assam, India

1. Introduction

An Euler diagram consists of a collection of some simple closed curves in a plane that represents some sets. Euler diagrams have a wide variety of applications ranging from information visualization to logical reasoning. A lot of research work has been performed by many researchers on these diagrams. Stapelton et al.[1] have provided a general definition of Euler diagrams and discussed its various properties. J. Flower et al.[2] have developed various methods for the generation of Euler diagrams. Each method generates a particular class of Euler diagrams with a set of properties called wellformedness conditions. J. House et al.[3] have developed an algorithm to generate an Euler diagram from an abstract description of the definition. An abstract description specifies the intersection between the curves. One can construct the dual and closed curves using the abstract description.

Stapleton et al. have shown how a new Euler diagram layout can be developed by modifying the existing layout of the diagram having the same set of wellformedness properties. A new curve can be added to the existing layout by modifying the dual of the Euler graph which forms a new layout[4][5]. The properties of an Euler diagram can be altered by making some changes on the vertex label graph generated from the abstract description of the Euler diagram. Rodgers et al.[6] have discussed how a number of Euler diagrams having the same or different sets of properties can be generated by making some changes in the Euler graph. One can choose a diagram having a particular set of properties from these Euler diagrams.

Though a lot of research has been performed on Euler diagrams, not much have been performed on the generation of Euler graphs from Euler diagrams providing ample scope for research in this area too. Only recently Pathak et al.[7] have presented a discussion on the construction of a particular class of Euler graphs from an Euler diagram with a certain set of properties. Various properties of the corresponding Euler graph, dual graph and the intersection graph have been studied and various theoretical properties of these graphs also have been discussed [7].

In this paper we have discussed the generation of a particular class of Euler graphs from a particular Euler diagram having a certain set of properties. We have studied the various properties of the corresponding dual graph of the Euler graph and the intersection graph generated from the Euler diagram. We have also discussed the various theoretical properties of these graphs.

2. Generation of Euler graph

We consider the Euler diagram shown in Figure1. This Euler diagram consists of five simple closed curves A, B, C, D and E with intersections between the curves A, B, E and C, D, E . Curve E intersects both the intersecting pairs of curves A, B and C, D.

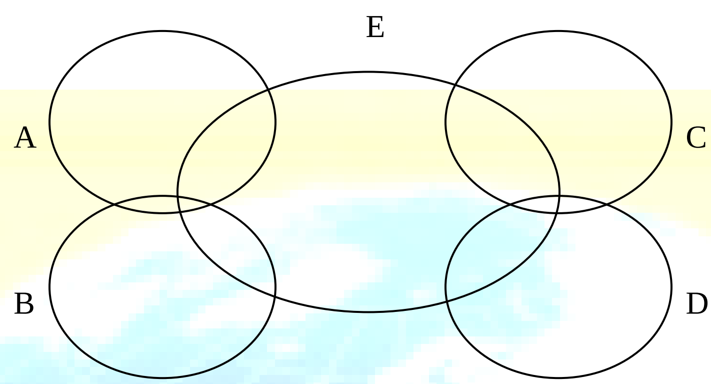

Figure 1: A special pattern of Euler diagram.

Now, the corresponding Euler graph of this Euler diagram can be constructed by considering a vertex for each point in the diagram where two curves intersect as shown in Figure 2. The edges connecting the vertices of the graph correspond to the curve segments in the diagram between the intersection points and the constructed graph, thus obtained from the Euler diagram of figure 1 is shown in figure 2.

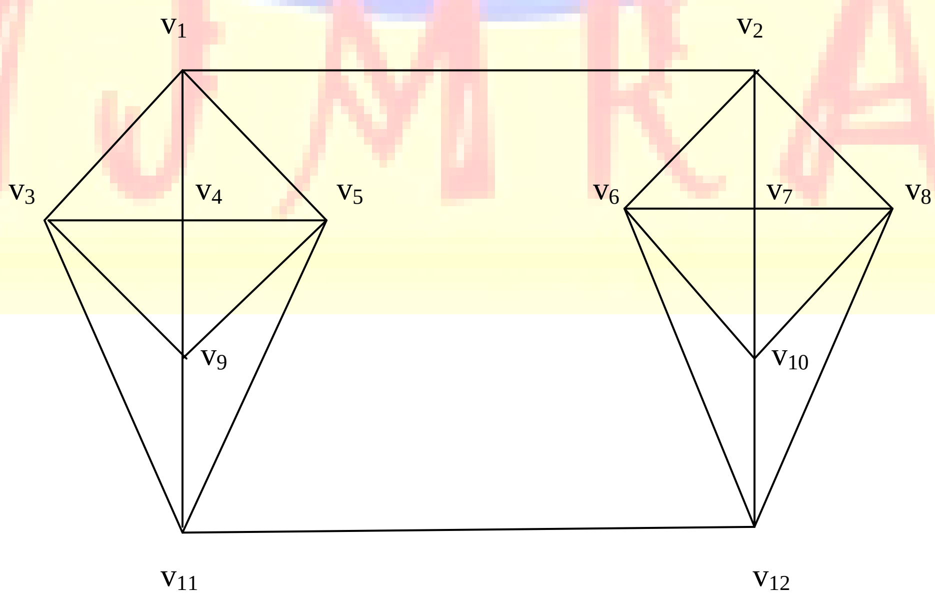

Figure 2: Euler graph of the Euler diagram in figure 1.

Again if the Euler diagram in figure 1 is expanded horizontally by adding another curve F intersecting the curves E, C and D then the diagram becomes as shown in figure 3.

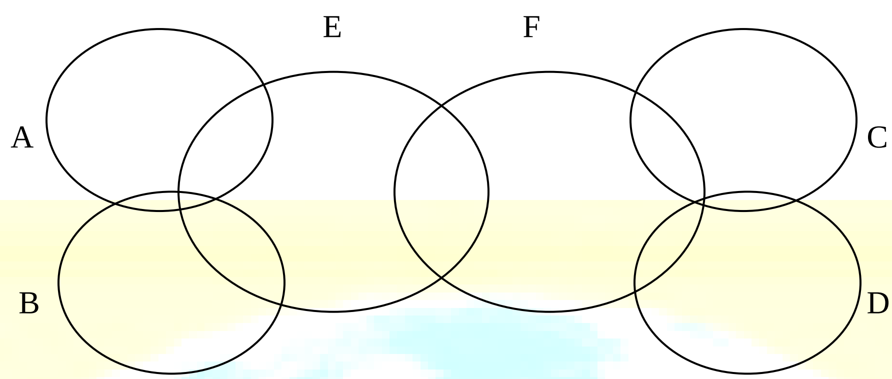

Figure 3: Euler diagram obtained by horizontal expansion of the diagram in figure 1.

Now considering the same construction process of the Euler graph as discussed above for the graph of figure 2, we have an Euler graph as shown in figure 4.

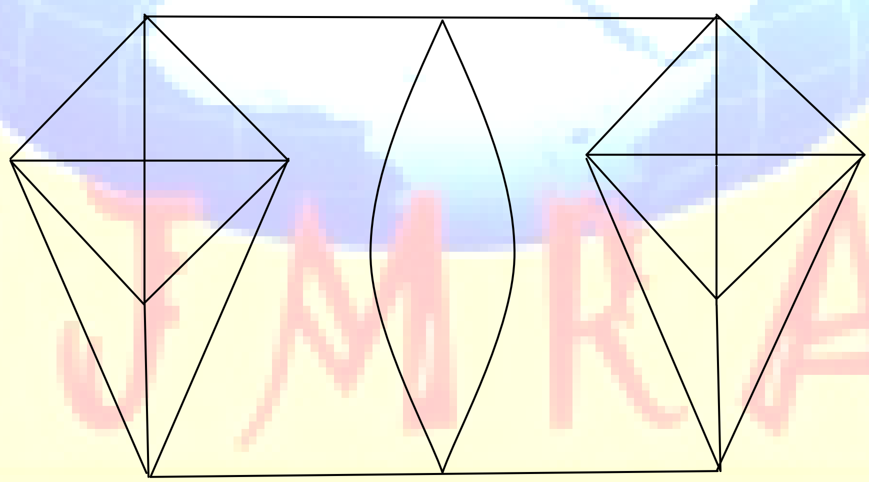

Figure 4: Euler graph of the Euler diagram of figure 3.

Continuing the process of construction of the Euler diagram by expanding the diagram horizontally, i.e. by adding more closed curves to the existing Euler diagram as shown in figure 3 above, we obtain a class of the corresponding Euler graphs G(2n+10, 4n+20) for n≥1 which is a regular graph of degree four.

If we construct the corresponding dual graphs for each of the Euler graph of the class G(2n+10, 4n+20) for n≥1 we observe that all the duals are of a particular structure and hence belong to a class H(2n+12, 4n+20) for n≥1. The dual graph of the Euler graph in figure 2, i.e., for n=1, is shown in figure 5.

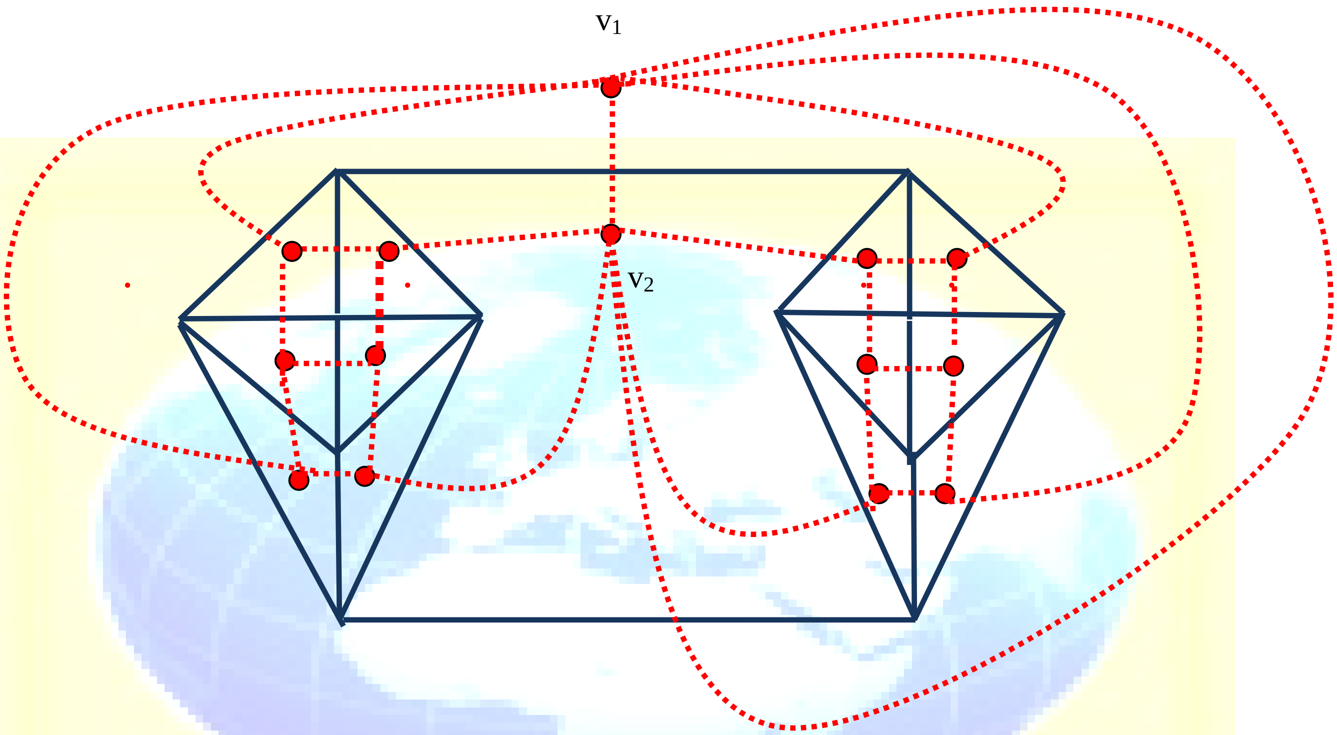

Figure 5: Dual of the Euler graph in figure 2.

Now, in the graph G(2n+10, 4n+20) for n >1, if the parallel edges are deleted and the vertices in series are merged, then the Euler graph takes the shape as shown in figure 2.

We can construct the intersection graph which has been discussed by Simonetto[8] for the Euler diagram of figure 3 and this is shown in figure 6.

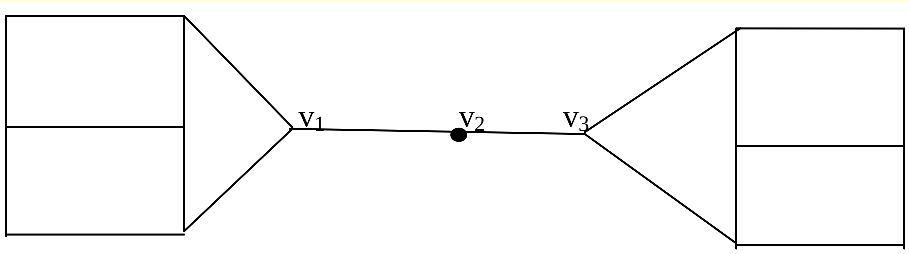

Figure 6: Intersection graph of the Euler diagram in figure 3.

Continuing the process of construction of the corresponding intersection graphs from the pattern of the Euler diagrams, we observe that the intersection graphs are of the form $G^{/}(2n+11, 2n+18)$ for $n\geq1$. We have also observed that these intersection graphs are always 3 colorable.

3. Theoretical discussions

Theorem 3.1: The graph G(2n+10, 4n+20) for $n\geq1$ is a regular planar graph of degree 4 and hence its dual exists.

Proof: From the construction process of the graph G(2n+10, 4n+20) for $n\geq1$, we have all the degrees of the graph as equal and the degree of each vertex is found to be four. Hence the graph is regular of degree four.

Theorem 3.2: The graph G(2n+10, 4n+20) for $n\geq1$ is 3-colorable.

Proof: From the construction process of the graph G(2n+10, 4n+20) for $n\geq1$ in figure 2 we observe that the graphs always contain some triangles. We know that a graph containing a triangle is always 3-colorable. Hence, the proof.

Theorem 3.3: The dual H(2n+12, 4n+20) for $n\geq1$ of the graph G(2n+10, 4n+20) for $n\geq1$ is not an Euler graph and is 2-colorable.

Proof: We know that in an Euler graph, all the vertices are of even degree. In figure 5, we observe that the dual H has some vertices of odd degree. Hence the dual H of the graph G is not an Euler graph.

Now we are to prove that the dual H is 2-colorable. We know that a planar graph always has some regions. Thus, it would be sufficient to prove that the dual is 2-colorabe if we can prove that the graph G itself is 2-colorable.

We observe that the graph G(2n+10, 4n+20) is a 4-reugular graph for n=1 and it has fourteen regions R_1, R_2,……,R_{14}.

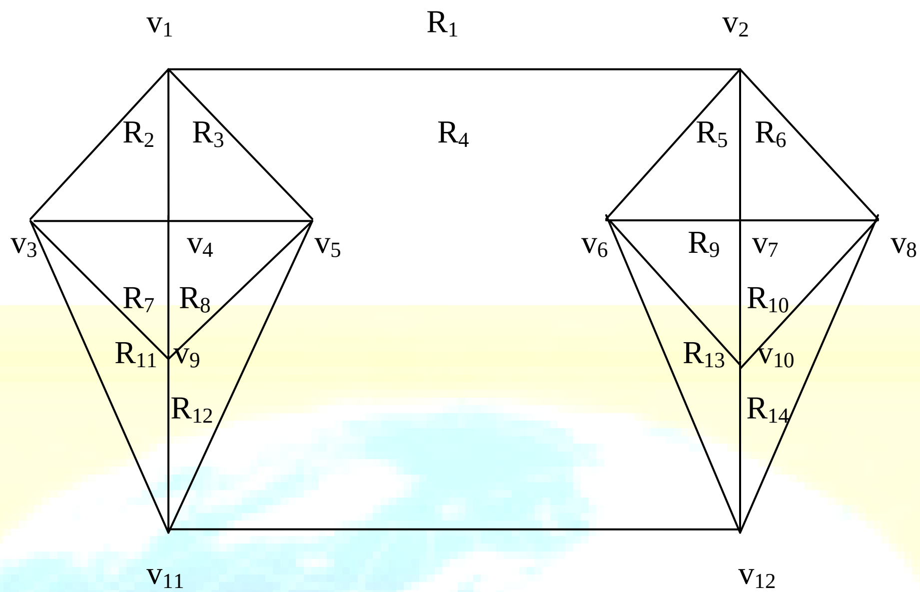

Figure 7: Showing the regions of the Euler graph in figure 2.

Let us consider the vertex v_1 of the graph as shown in figure 7. The degree of v_1 is four. Thus, there are four regions R_1, R_2, R_3 and R_4 around v_1. Now, considering a point from each of these regions and connecting the points by an edge if there is a common boundary among the regions we get a graph as shown in figure 8. This graph is a cycle having four vertices. Thus, this graph is 2-colorable.

Figure 8: Studying the colors of the regions around vertex v_1 in figure 7.

Figure 9: Studying the colors of the regions around vertex v_2 in figure 7.

Similarly, let us consider the vertex v_2 in figure 7. The degree of v_2 is also four and it is also surrounded by four regions R_1, R_4, R_5 and R_6. Performing the same procedure discussed above for vertex v_1 we get another graph as shown in figure 9. This graph also has a cycle of four vertices. Thus, this graph is also 2-colorable.

In figure 7, we also observe that there is no common boundary between the regions R_1 and R_3, R_2 and R_4, R_1 and R5, R_4 and R_6. Thus, the regions around the vertices v_1 and v_2 can be properly colored by two different colors. In figure 7 we observe that the faces of the graph are 2-colorable. Therefore, the dual of the graph in figure 7 is also 2-colorable and it can be properly colored by two colors c_1 and c_2 as shown in figure 10.

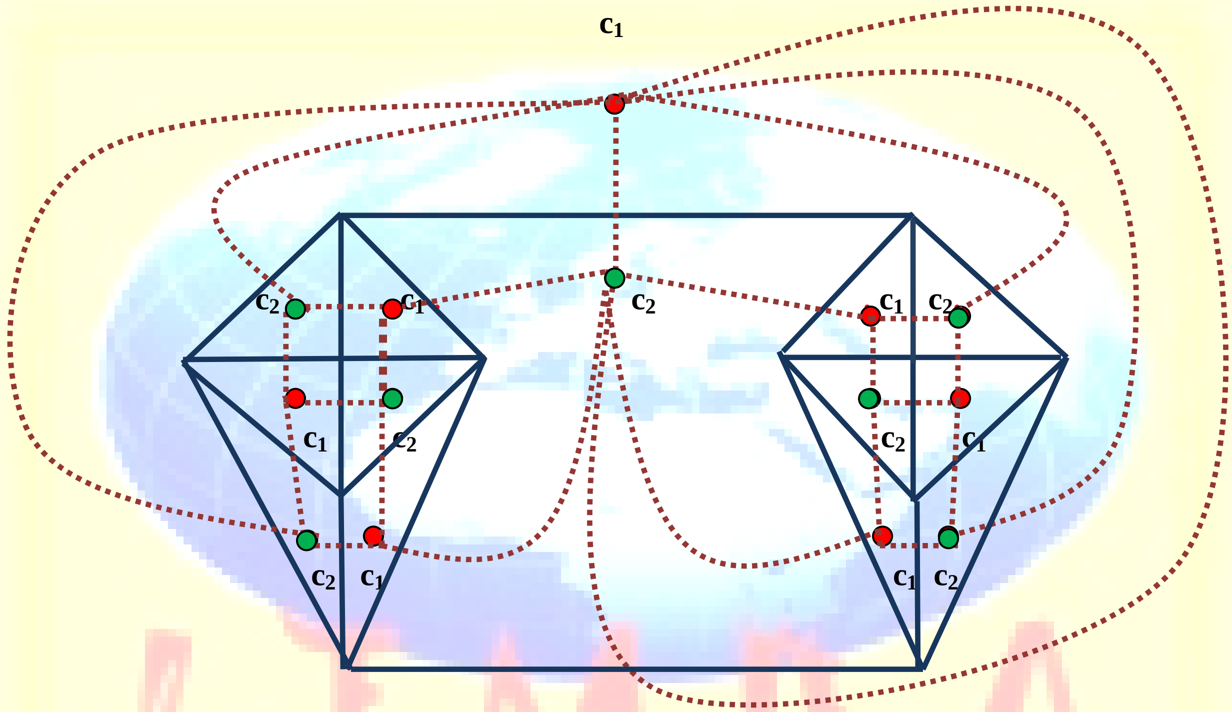

Figure 10: Coloring the dual of the Euler graph in figure 7.

Let us suppose that the result is true for n=k. Hence the graph H(2n+12, 4n+20) is 2-colorable. We observe that every time n is incremented by one, we always get a 4-regular graph, i.e., each vertex is of degree four and surrounded by four regions. For this graph, we have a cycle with even(four) number of vertices. The resulting cycles can be colored by two different colors as discussed above. Therefore, the dual of the graph is 2-colorable. Hence the proof.

Theorem 3.4: The graph G(2n+10, 4n+20) for n≥1 always has a 2-factor.

Proof: We know that any 2k regular graph is 2 factorable for some integer k. Now, the graph G(2n+10, 4n+20) for n≥1 is a regular graph of degree 4. Hence, 2k=4 i.e., k=2. Hence, the proof.

Theorem 3.5: The intersection graph $G^{/}$(2n+11, 2n+18) for n≥1 is not an Euler graph and is 2-colorable.

Proof: We know that in an Euler graph, all the vertices are of even degree. In figure 6, we observe that the intersection graph $G^{/}$ has some vertices of odd degree. Hence the intersection graph $G^{/}$ obtained from the Euler diagram in figure 1 is not an Euler graph.

Similar to the proof of theorem 3.3, it can be proved that intersection graph $G^{/}$ is also 2-colorable.

Theorem 3.6: The graph G(2n+10, 4n+20) for n≥1 is 2-connected.

Proof: In figure 2, we observe that every path between the vertices v_3 and v_8 must pass through either vertex v_1 or v_{11}. Hence, removal of both the vertices v_1 and v_{11} will eliminate every path between the vertices v_3 and v_8. Thus, removal of the vertices v_1 and v_{11} leaves the graph G(2n+10, 4n+20) for n≥1 disconnected into two separate components.

Theorem 3.7: The dual H(2n+12, 4n+20) for n≥1 of the graph G(2n+10, 4n+20) for n≥1 is 2-connected.

Proof: Similar to the proof of theorem 3.6 we observe that in figure 5, removal of the vertices v_1 and v_2 disconnects the dual H(2n+12, 4n+20) for n≥1 of the graph G(2n+10, 4n+20) for n≥1 into two components. Thus, the dual H is 2-connected.

Theorem 3.8: The intersection graph $G^{/}$(2n+11, 2n+18) for n≥1 is minimally connected i.e., 1-connected.

Proof: In figure 6, we observe that removal of any one of the vertices v_1, v_2 or v_3 disconnects the graph into two separate components. Thus, the intersection graph $G^{/}$(2n+11, 2n+18) for n≥1 obtained from the Euler diagram in figure 1 is minimally connected.

4. Conclusion

The construction process of an Euler graph from a special pattern of Euler diagrams have been focused in this paper. Some important properties of the Euler graph have also been discussed in the paper. Two very important findings discussed in the paper are that the dual and the intersection graph of the Euler graph obtained from the special pattern of Euler diagrams are 2-colorable.

5. References

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